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Research Article

A Variable Step-Size Proportionate Affine Projection Algorithm for Identification of Sparse Impulse Response

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Proportionate adaptive algorithms have been proposed recently to accelerate convergence for the identification of sparse impulse response. When the excitation signal is colored, especially the speech, the convergence performance of proportionate NLMS algorithms demonstrate slow convergence speed. The proportionate affine projection algorithm (PAPA) is expected to solve this problem by using more information in the input signals. However, its steady-state performance is limited by the constant step-size parameter. In this article we propose a variable step-size PAPA by canceling the a posteriori estimation error. This can result in high convergence speed using a large step size when the identification error is large, and can then considerably decrease the steady-state misalignment using a small step size after the adaptive filter has converged. Simulation results show that the proposed approach can greatly improve the steady-state misalignment without sacrificing the fast convergence of PAPA.

1. Introduction

Adaptive filtering algorithms can find application in many real-world systems [1–3], such as wireless channel equalizers, echo cancelers, noise reduction, and speech enhancement, for example, an echo canceler is designed to identify an unknown echo path. Its output is a replica of the echo signal, which is then removed from the near-end signal to achieve echo cancellation. Nowadays, echo path is becoming longer and longer with the increased demand for higher-quality communication, especially for voice-over IP systems. For a network echo canceler, the number of coefficients varies from 512 to 2048 in order to deal with a total delay greater than 64 milliseconds [4]. Conventional adaptive algorithms, such as the least mean square (LMS) algorithm and the normalized LMS (NLMS) algorithm [2, 5], suffer severely from slow convergence with this kind of long filter, especially for colored signals. Much effort has been made to design new algorithms to improve the convergence speed of adaptive filters with hundreds or thousands of coefficients.

A new kind of proportionate adaptive filtering algorithm has received much attention recently [6–8]. Proportionate adaptive algorithms are based on the fact that most long impulse responses are *sparse* in nature because only a small percentage of coefficients are active and most of the others are zeros. Conventional adaptive algorithms assign the same step size to all coefficients. As a result, large coefficients require many more iterations to converge than small ones. To accelerate the convergence of the large coefficient, it seems that we should assign them larger step size than that of small ones, which will yield proportionate adaptation. The idea behind proportionate adaptive algorithms is to update each coefficient of the filter individually by assigning each coefficient a step size proportionate to its estimated magnitude. Various proportionate adaptive algorithms have been proposed to exploit this sparse structure. Their convergence speeds are greatly improved [7] over conventional adaptive algorithms. The proportionate NLMS (PNLMS) algorithm was firstly proposed in [9]. It greatly speeds up the initial convergence of adaptive filters. However, its convergence begins to slow dramatically thereafter. Many modifications have been proposed to improve it, such as the PNLMS++ algorithm [10], the IPNLMS algorithm [11], the CPNLMS algorithm [12], the improved IPNLMS algorithm [13], the IPMDF algorithm [14], and the mu-law PNLMS (MPNLMS) algorithm [15]. Among these variants, the

MPNLMS algorithm is one of the fastest in the framework of proportionate adaptation. Instead of using magnitude directly, the logarithm of the magnitude is used as the step gain of each coefficient, so the MPNLMS algorithm can consistently converge to the steady state for sparse impulse response. The SPNLMS algorithm [15] is proposed to reduce the heavy computational complexity of MPNLMS without loss of fast convergence.

The step-size control matrix of MPNLMS was derived on the assumption that the input is white. For colored input signals, especially speech, convergence speed will depend on the eigenvalues of the input signal's autocorrelation matrix. The proportionate affine projection algorithm (PAPA) is the natural extension of the PNLMS algorithm. It is expected to present faster convergence for highly correlated input signals at the cost of a modest increase in computational complexity.

Besides convergence speed, another important aspect of the adaptive algorithm is its steady-state performance: low misalignment is desirable. Unfortunately, the design of the step-size control matrix cannot decrease the misalignment in the framework of proportionate adaptation. The steady-state misalignment of the proportionate algorithms is approximately equal to that of their nonproportionate counterparts [9]. We know that the step-size parameter reflects a tradeoff between fast convergence and low misalignment. When step size is adjusted to obtain faster convergence, misalignment becomes larger, and vice versa. If we adaptively control the step size to be large in the transient state and to be small as convergence proceeds, both fast convergence and low misalignment can be achieved. Different adaptive step-size control approaches have been proposed and studied in literature relating to this concept. In [16], the squared instantaneous error was exploited as a criterion to change the step size. In [17], an optimal step size was proposed for NLMS algorithm by minimizing the mean-square deviation at each iteration. A variable step-size NLMS algorithm was described in [18] to improve the estimation of the power level of the disturbance signals. It was used to decide the optimal step size at each iteration. In [19], a steepest descent method was proposed to adaptively update the step size to minimize squared error. By combining the input vector and the instantaneous error vector, a variable step-size approach was proposed for APA in [20]. A nonparametric variable step-size NLMS algorithm, NPVSS-NLMS, was proposed in [21] by adjusting the step size to cancel a posteriori error. Recently, this approach was applied in the undermodeling acoustic echo cancellation system [22, 23]. It was further extended to APA with a new perspective of signal enhancement in [24]. As can be seen, these approaches are only applicable to nonproportionate adaptive algorithms.

In this article, we propose a variable step-size proportionate affine projection algorithm (VSS-PAPA) for the identification of sparse impulse response. Theoretically, in a noise free environment, PAPA has optimal convergence speed and zero misalignment by canceling the a posteriori output estimation error at each iteration. However, with the presence of a disturbance signal, canceling the a posteriori estimation error will introduce additional noise into the coefficient update [21]. Taking the effect of background noise into account, we derive a PAPA with variable step size parameter to cancel a posteriori estimation error at each iteration. The variable step size is large when the adaptive filter is in its transient state. Hence, it converges fast. Then the step size becomes small when the adaptive filter reaches the steady state, so misalignment is significantly decreased. The proposed algorithm demonstrates excellent performance by combining the fast proportionate algorithm with variable step-size technique for identification of sparse impulse response.

The organization of this article is as follows. In Section 2, we briefly overview the proportionate affine projection algorithm and various definitions of the step-size control matrix of proportionate adaptation. In Section 3, a variable step size approach is proposed for PAPA to achieve better performance. In Section 4, many computer simulation results are presented to illustrate the excellent performance of the proposed algorithm. Finally, Section 5 concludes our research.

2. Overview of Proportionate Adaptive Algorithms

Consider a system identification problem. Bold lowercase letters indicate vectors and bold uppercase letters denote matrices. All vectors are column vectors, $(\cdot)^T$ indicates transpose, and t is the time index. Also, $\mathbf{w}_{\text{opt}}$ is an unknown sparse impulse response and $\mathbf{w}$ is an adaptive filter. The length of $\mathbf{w}_{\text{opt}}$ and $\mathbf{w}$ is supposed to be same, N. The input vector $\mathbf{x}(t) = [x(t)\,x(t-1)\ldots x(t-N+1)]^T$, the output of the adaptive filter $y(t) = \mathbf{w}_{\text{opt}}^T\mathbf{x}(t)$, and the desired signal $d(t) = y(t) + v(t)$, where $v(t)$ is a disturbance signal, which may be background noise or/and measurement noise.

The APA achieves a faster convergence speed for correlated input signals than the NLMS algorithm with only a modest increase in computational complexity. It exploits more information from the input signal, not only the current input vector but also the most recent P input vectors. The proportionate APA (PAPA) is expected to converge faster than the proportionate NLMS algorithms for colored input signals. Define P as the projection order, the input matrix as the P successive input vector, $\mathbf{X}(t) = [\mathbf{x}(t)\,\mathbf{x}(t-1)\,\ldots\,\mathbf{x}(t-P+1)]$, and the desired vector as the P successive past value of $d(t)$, $\mathbf{d}(t) = [d(t)\,d(t-1)\,\cdots\,d(t-P+1)]^T$. The error vector $\mathbf{e}(t)$ can be written as

$$\mathbf{e}(t) = \mathbf{d}(t) - \mathbf{X}^T(t)\mathbf{w}(t). \tag{1}$$

The PAPA can be briefly summarized as follows:

$$\mathbf{w}(t+1) = \mathbf{w}(t) + \alpha\mathbf{G}(t)\mathbf{X}(t)\left[\mathbf{X}^T(t)\mathbf{G}(t)\mathbf{X}(t) + \varepsilon\mathbf{I}\right]^{-1}\mathbf{e}(t), \tag{2}$$

$$\mathbf{G}(t) = \operatorname{diag}\{g_0(t), g_1(t), \ldots, g_{N-1}(t)\}, \tag{3}$$

where α is a overall constant step size, ϵ is the regularization parameter, and $\mathbf{I}$ is a $P \times P$ identity matrix. The definition of the diagonal element of matrix $\mathbf{G}(t)$ can be summarized as

$$L_{\max} = \max\left\{\delta_\rho, F(w_0(t)), \ldots, F(w_{N-1}(t))\right\}, \tag{4}$$

$$\gamma_n(t) = \max\{F(w_n(t)), \rho L_{\max}\}, \tag{5}$$

$$g_n(t) = \frac{\gamma_n(t)}{(1/N)\sum_{i=0}^{N-1}(t)}. \tag{6}$$

Here, F is a real-valued function to map the current coefficient estimate into a certain value of the proportionate step-size parameter; δ_ρ is used to prevent $\mathbf{w}(t)$ from stalling at the beginning, and has a typical value of 0.01; ρ is used to prevent the very small coefficients from stalling, and has typical values in the range from $1/N$ to $5/N$ [9]. Note that when $P = 1$, the PAPA degenerates into the PNLMS algorithm, and when all of the elements of $\mathbf{G}(t)$ are identical, that is, $g_0(t) = \cdots = g_{N-1}(t) = 1$, the PAPA reduces to the standard APA.

The PNLMS algorithm [9] has proposed a simple function as $F(w_n(t)) = |w_n(t)|$. It has very fast initial convergence speed. However, its convergence slows thereafter. Furthermore, its convergence speed degrades greatly if the target impulse response is not sparse enough. The MPNLMS algorithm proposed in [15, 25] achieves the fastest step size control matrix $\mathbf{G}(t)$ in the proportionate adaptation framework. Instead of using the absolute value of the coefficient magnitude directly, its logarithm is used as the step size. Hence, both large and small coefficients converge at the same rate, so that the overall convergence speed of the adaptive filter is greatly accelerated. For MPNLMS, $F(w_n(t)) = \ln(1 + \mu|w_n(t)|)$, where μ is an objective convergence criterion, typically $\mu = 1000$. Many simulation results have proved that the MPNLMS algorithm is one of the fastest proportionate algorithms [26]. The main disadvantage of MPNLMS is its heavy computation cost because of the presence of N logarithmic operations in every iteration. A line segment is proposed to approximate the mu-law function, which leads to a computation efficient algorithm, SPNLMS [25], where

$$F(w_n(t)) = \begin{cases} 400|w_n(t)|, & |w_n(t)| < 0.005, \\ 2, & \text{otherwise.} \end{cases} \tag{7}$$

The step-size control matrix defined by MPNLMS was derived on the assumption that the input is white. The mu-law PAPA (MPAPA) is expected to achieve faster convergence speed than MPNLMS for colored input signals. Its computation efficient version, SPAPA, is favorable for real-world application because of its implementable low computational complexity.

3. Variable Step-Size Proportionate Affine Projection Algorithms

3.1. Algorithm Formulation. Our objective is to find a variable step-size approach that is applicable to PAPAs. Unfortunately, because of the presence of $\mathbf{G}(t)$, it is very difficult to analyze the transient performance of PAPAs. In this section, we propose a variable step size for PAPA.

The APA can be derived from the principle of least perturbation, that is, to maintain the next coefficient vector as close as possible to the current estimate, while forcing the a posteriori output estimation error to be zeros [2, 5, 27]. The a posteriori output estimation error vector $\mathbf{r}(t)$ is defined as [5]

$$\begin{aligned} \mathbf{r}(t) &= \mathbf{d}(t) - \mathbf{X}^T(t)\mathbf{w}(t+1) = \mathbf{X}^T(t)\widetilde{\mathbf{w}}(t+1) + \mathbf{v}(t), \\ \widetilde{\mathbf{w}}(t) &= \mathbf{w}_{\text{opt}} - \mathbf{w}(t), \end{aligned} \tag{8}$$

where $\widetilde{\mathbf{w}}(t)$ is the coefficient error vector and $\mathbf{v}(t) = [v(t)\, v(t-1) \cdots v(t-P+1)]^T$ is the disturbance signal vector. Compared to $\mathbf{r}(t)$, the error $\mathbf{e}(t)$ in (1) plays the role of the a priori output estimation error vector.

The APA can satisfy the principle of least perturbation in a noise-free system. It has the fastest convergence speed and zero misalignment by canceling $\mathbf{r}(t)$ at each iteration. The optimal step size is *one* in this case. However, in practical application, a disturbance signal is inevitable. Therefore, the adaptive algorithm cannot achieve zero misalignment. This could be explained by the fact that in the presence of $v(t)$, attempts to force $\mathbf{r}(t)$ to be zero will introduce noise to the adaptive filter update [21]. Actually, what we would like is to force the a posteriori estimation error to be zero. That is

$$\mathbf{X}^T(t)\widetilde{\mathbf{w}}(t+1) = \mathbf{0}, \tag{9}$$

where $\mathbf{0}$ is a $P \times 1$ column vector whose elements are all zeros. Combining (8) with (9) implies that in a noisy environment we should update the coefficients to make the a posteriori error not to be zero, but to be the disturbance signal: $\mathbf{v}(t)$,

$$\mathbf{r}(t) = \mathbf{v}(t). \tag{10}$$

In practical application, although the disturbance signal $v(t)$ is not available, its power level can be estimated. For this reason, the optimal step-size parameter can be found in such a way that

$$E\left\{r_p^2(t)\right\} = E\left\{v_p^2(t)\right\}, \quad p = 0 \cdots P-1, \tag{11}$$

where $r_p(t)$ is the pth element of $\mathbf{r}(t)$, and $v_p(t)$ is the pth element of $\mathbf{v}(t)$. Note that $v_p(t) = v(t-p)$.

Based on above notion, a VSS-PAPA can be derived as follows. Rewrite (2) with a $P \times P$ time-varying step-size diagonal matrix $\boldsymbol{\alpha}(t)$, ignoring the regularization term $\epsilon\mathbf{I}$, :

$$\mathbf{w}(t+1) = \mathbf{w}(t) + \mathbf{G}(t)\mathbf{X}(t)\left[\mathbf{X}^T(t)\mathbf{G}(t)\mathbf{X}(t)\right]^{-1}\boldsymbol{\alpha}(t)\mathbf{e}(t). \tag{12}$$

Subtracting $\mathbf{w}_{\text{opt}}$ at both sides and rearranging the terms, we get

$$\widetilde{\mathbf{w}}(t+1) = \widetilde{\mathbf{w}}(t) - \mathbf{G}(t)\mathbf{X}(t)\left[\mathbf{X}^T(t)\mathbf{G}(t)\mathbf{X}(t)\right]^{-1}\boldsymbol{\alpha}(t)\mathbf{e}(t). \tag{13}$$

Premultiplying $\mathbf{X}^T(t)$ at both sides yields a relation between the a posteriori estimation error and the a priori output estimation error:

$$r_p(t) = \left[1 - \alpha_p(t)\right] e_p(t), \tag{14}$$

where $\alpha_p(t)$ is the pth diagonal element of $\boldsymbol{\alpha}(t)$, and $e_p(t)$ is the pth element of $\mathbf{e}(t)$. This result is very interesting in relation to PAPA. It can be observed that the a posteriori estimation error $\mathbf{r}(t)$ is determined by the step size parameter $\boldsymbol{\alpha}(t)$ and error vector $\mathbf{e}(t)$ and is independent from $\mathbf{G}(t)$. Consequently, a simple variable step-size approach is expected for PAPA from this relation, following a procedure similar to [24].

Squaring and taking mathematical expectation at both sides of (14), and combining it with (11), give

$$E\left\{r_p^2(t)\right\} = \left[1 - \alpha_p(t)\right]^2 E\left\{e_p^2(t)\right\} = E\{v^2(t-p)\}. \tag{15}$$

Solving the equation, the pth time-varying step-size $\alpha_p(t)$ is obtained with a simple expression as

$$\alpha_p(t) = 1 - \sqrt{\frac{\sigma_v^2(t-p)}{\sigma_{e_p}^2(t)}}, \tag{16}$$

where $\sigma_v^2(t-p) = E\{v^2(t-p)\}$ is the variance of $v(t-p)$ and $\sigma_{e_p}^2(t) = E\{e_p^2(t)\}$ is the variance of $e_p(t)$.

In the transient state of the adaptive filter, $\sigma_{e_p}^2(t)$ will be large, hence $\alpha_p(t)$ is also large. Consequently, fast convergence speed can be expected. After the adaptive filter reaches to within the immediate vicinity of its optimal value, $\sigma_{e_p}^2(t)$ becomes small, hence $\alpha_p(t)$ decreases. As a result, low misalignment can be observed.

There are some practical considerations related to this expression. The first is the estimations of $\sigma_{e_p}^2(t)$ and $\sigma_v^2(t)$. The quantity of $\sigma_{e_p}^2$ can be estimated using an exponential window as

$$\hat{\sigma}_{e_p}^2(t) = (1 - \lambda_1)\hat{\sigma}_{e_p}^2(t-1) + \lambda_1 e_p^2(t), \tag{17}$$

where

$$\lambda_1 = 1 - \frac{1}{K_1 N}, \quad (K_1 \in Z^+, K_1 \geq 1). \tag{18}$$

A large K_1 can obtain a smooth estimate of $\sigma_{e_p}^2(t)$ but it will reduce the tracking ability of the adaptive filter. In practical application, power estimation of the disturbance signal, $\hat{\sigma}_v^2(t)$, can be obtained during the silences in a network echo cancellation system. An estimate of the disturbance signal, $\hat{v}(t)$, can even be obtained using an additional adaptive filter, as proposed in [18]. Therefore, by using the same method with $\hat{\sigma}_{e_p}^2(t)$, $\hat{\sigma}_v^2(t)$ can be obtained by

$$\hat{\sigma}_v^2(t) = \lambda_1 \hat{\sigma}_v^2(t-1) + (1 - \lambda_1)\hat{v}^2(t). \tag{19}$$

The second issue is stability. These estimates could lead to minor deviations from their theoretical values, which may result in a negative step size or a large one and force the adaptive algorithm to diverge. It is necessary to restrict $\alpha_p(t)$ in range so that the stability of the adaptive algorithm is guaranteed, $0 \leq \alpha_{\min} \leq \alpha_p(t) \leq \alpha_{\max} \leq 2$. Suitable choice of $\alpha_{\min}$ and $\alpha_{\max}$ can make the proposed algorithm robust to an inaccurate estimate of $\hat{\sigma}_v^2$. More detailed discussions on this issue will be presented in the following subsection.

The third issue is the determination of $\mathbf{G}(t)$. Although it can be determined by any proportionate adaptive algorithm, it is preferable to adopt the segment proportionate version described in (3)–(6) and (7) because it has excellent performance and light computation load. We will use this definition throughout this article. The proposed algorithm with this definition of $\mathbf{G}(t)$ will be referred as VSS-SPAPA for convenience sake. Note that when $\mathbf{G}(t)$ is an identity matrix the proposed VSS-SPAPA degrades into the VSS-APA in [24].

It seems that the proposed variable step-size PAPA is similar to the set-membership PAPA (SM-PAPA) proposed in [28], where $\alpha_p(t)$ is replaced by

$$\alpha_{sm,p}(t) = \begin{cases} \dfrac{1-\gamma}{\left|e_p(t)\right|}, & \text{if } \left|e_p(t)\right| > \gamma, \\ 0, & \text{otherwise.} \end{cases} \tag{20}$$

Here, γ is a predetermined parameter as a bound on the noise. Since there is no averaging on, it is obvious that we cannot expect a lower misalignment than we propose in this article. The proposed variable step size described in (16) provides an optimal criteria of γ for the SM-PAPA, that is, γ should choose according to σ_v.

3.2. Adaptive Estimate of $\sigma_v^2(t)$ and Its Influence. The proposed VSS-SPAPA can obtain optimal $\alpha_p(t)$ if an accurate estimate of $\sigma_v^2(t)$ is available. As explained in the previous subsection, a relatively accurate estimate can be easily obtained during silence in a network echo cancellation system, since there are many pauses in natural speech. However, if the power of the disturbance signals changes between two consecutive estimations, its new estimate will not be available immediately. A method was proposed to adaptively estimate $\sigma_v^2(t)$ only using the signals available in the system, which can be described by [24]

$$\begin{aligned} \hat{\sigma}_v^2(t) &= \max\left\{0, \hat{\sigma}_d^2(t) - \hat{\sigma}_{\hat{y}}^2(t)\right\}, \\ \hat{\sigma}_d^2(t) &= \lambda_2 \hat{\sigma}_d^2(t-1) + (1-\lambda_2) d^2(t), \\ \hat{\sigma}_{\hat{y}}^2(t) &= \lambda_2 \hat{\sigma}_{\hat{y}}^2(t-1) + (1-\lambda_2)\hat{y}^2(t), \end{aligned} \tag{21}$$

where $\hat{y}(t) = \mathbf{x}^T(t)\mathbf{w}(t)$ is the output of the adaptive filter, and

$$\lambda_2 = 1 - \frac{1}{K_2 N}, \quad (K_2 \in Z^+, K_2 \geq K_1). \tag{22}$$

This estimate is approximately accurate after the adaptive filter has reached its steady state, because

$$E\{v^2(t)\} \approx E\{d^2(t)\} - E\{\hat{y}^2(t)\}. \tag{23}$$

The method provides a suboptimal solution of $\sigma_v^2(t)$ if it is unavailable in the given practical application. However, after the adaptive filter has converged to within the immediate vicinity of its optimal value, it can also be found that [22]

$$E\{e^2(t)\} \approx E\{d^2(t)\} - E\{\hat{y}^2(t)\}. \qquad (24)$$

Therefore, the difference between $\hat{\sigma}^2_{e_p}(t)$ and $\hat{\sigma}^2_v(t)$ will be insignificant in the steady state of the adaptive filter. As a result, $\alpha_p(t)$ will be very small, and low steady-state misalignment observed. However, this approach will lead to a slow convergence speed if this approach is applied during the transient period of the adaptive filter, for example, if the unknown impulse response changes suddenly, this approach is believed to present poor tracking ability because $\alpha_p(t)$ remains relatively small. To improve the adaptive filter's tracking ability in this scenario, the value of $\alpha_{\min}$ should not be too small, and the steady-state misalignment will be increased as a consequence. An alternative is to introduce a impulse response variation detector in the algorithm, see [24] and reference therein.

Let us discuss the influence of an inaccurate estimate of $\hat{\sigma}^2_{\hat{v}}(t)$ on the convergence performance of the adaptive algorithm. In the case of $\hat{\sigma}^2_v(t) \gg \sigma^2_v(t)$, $\alpha_p(t)$ will be smaller than its optimal value defined in (16). The convergence speed of the proposed algorithm will be slowed because $\alpha_p(t)$ reaches $\alpha_{\min}$ soon. However, low misalignment will be achieved using more iterations. In the case of $\hat{\sigma}^2_v(t) = \sigma^2_v(t)$, $\alpha_p(t)$ will be greater than its optimal value. Fast initial convergence speed will be observed but the misalignment will increase because $\alpha_p(t)$ is close to $\alpha_{\max}$. For a modest inaccurate estimate of $\hat{\sigma}^2_v(t)$, the performance of the proposed algorithm is better than that of the VSS-APA because it benefits from the fast proportionate adaptive algorithms. The simulation results in Section 4 show that the proposed algorithm can tolerate a large $\hat{\sigma}^2_v(t)$ estimate error.

The proposed variable step-size segment proportionate affine projection algorithm (VSS-SPAPA) is summarized in Algorithm 1 .

3.3. Computational Complexity. Compared to the standard APA, the additional computation load of the proposed VSS-SPAPA is composed of four parts. First, the calculation of $\mathbf{G}(t)$ costs $2N + 2$ multiplications or divisions, N additions, and $3N$ comparisons. Second, the calculation of $\mathbf{G}(t)\mathbf{X}(t)$ costs PN multiplications. Third, the estimate of $\hat{\sigma}^2_{e_p}(t)$ and calculation of $\boldsymbol{\alpha}(t)$ will cost $4P + 6$ multiplications or divisions, P square-root operations, and $2P + 2$ additions or subtractions. Finally, the calculation of $\boldsymbol{\alpha}(t)\mathbf{e}(t)$ costs P multiplication. The remaining operations are common with APA. In summary, the dominant additional computation cost of the proposed VSS-SPAPA is $(P + 2)N + 5P + 8$ multiplications or divisions operations and P square-root operations. For practical applications, such as network echo cancellation, the value of projection order P is usually in the range of 2–8. Therefore, the computational complexity of the proposed VSS-SPAPA is moderate. We propose that the additional computation load is worth the considerable performance improvement in sparse impulse response, and illustrate this in the following section.

Initialization:
$\mathbf{w}(0) = \mathbf{0}$; $\hat{\sigma}^2_v(0) = 0$; $\hat{\sigma}^2_{\hat{y}}(0) = 0$; $\hat{\sigma}^2_d(0) = 0$;
$\epsilon = PM\sigma^2_x$, $(M \in Z^+, M \ge 1)$
$\lambda_1 = 1 - 1/(K_1 N), (K_1 \in Z^+,\ K_1 \ge 1)$
$\lambda_2 = 1 - 1/(K_2 N), (K_2 \in Z^+,\ K_2 \ge K_1)$
for $p = 0, 1, \ldots, P - 1$
$\hat{\sigma}^2_{e_p}(t) = (1 - \lambda_1)\hat{\sigma}^2_{e_p}(t - 1) + \lambda_1 e^2_p(t)$;
For all t:

$$\bar{g}_n(t) = \begin{cases} 400|w_n(t)|, & |w_n(t)| < 0.005 \\ 2, & \text{otherwise.} \end{cases}$$

$L_{\max} = \max\{\delta_\rho, \bar{g}_0(t), \ldots, \bar{g}_{N-1}(t)\}$
$\gamma_n(t) = \max\{\bar{g}_n(t), \rho L_{\max}\}$
$g_n(t) = \gamma_n(t)/[1/N \sum_{i=0}^{N-1} \gamma_i(t)]$
$\mathbf{G}(t) = \operatorname{diag}\{g_0(t), g_1(t), \ldots, g_{N-1}(t)\}$
$\hat{\mathbf{y}}(t) = \mathbf{X}^T(t)\mathbf{w}(t)$
$\mathbf{e}(t) = \mathbf{d}(t) - \hat{\mathbf{y}}(t)$
if $\hat{\sigma}^2_v(t)$ is not available,
$\hat{\sigma}^2_d(t) = \lambda_2\hat{\sigma}^2_d(t - 1) + (1 - \lambda_2)d^2(t)$
$\hat{\sigma}^2_{\hat{y}}(t) = \lambda_2\hat{\sigma}^2_{\hat{y}}(t - 1) + (1 - \lambda_2)\hat{y}^2(t)$
$\hat{\sigma}^2_v(t) = \max\{0,\ \hat{\sigma}^2_d(t) - \hat{\sigma}^2_{\hat{y}}(t)\}$
for $p = 0, 1, \ldots, P - 1$
$\hat{\sigma}^2_{e_p}(t) = (1 - \lambda_1)\hat{\sigma}^2_{e_p}(t - 1) + \lambda_1 \hat{e}^2_p(t)$
$\alpha_p(t) = 1 - \sqrt{\hat{\sigma}^2_v(t - p)/\hat{\sigma}^2_{e_p}(t)}$
if $\alpha_p(t) < \alpha_{\min}$, $\alpha_p(t) = \alpha_{\min}$,
if $\alpha_p(t) > \alpha_{\max}$, $\alpha_p(t) = \alpha_{\max}$,
$\boldsymbol{\alpha}(t) = \operatorname{diag}\{\alpha_0(t), \ldots, \alpha_{P-1}(t)\}$
$\mathbf{w}(t + 1) = \mathbf{w}(t) + \mathbf{G}(t)\mathbf{X}(t)[\mathbf{X}^T(t)\mathbf{G}(t)\mathbf{X}(t) + \varepsilon\mathbf{I}]^{-1}\, \boldsymbol{\alpha}(t)\mathbf{e}(t)$

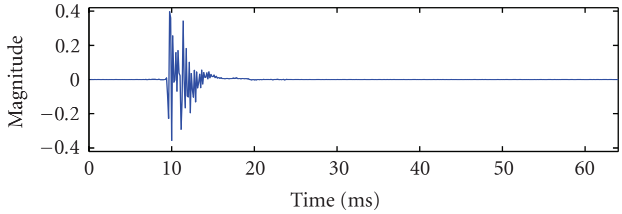
Algorithm 1: The proposed VSS-SPAPA.

4. Simulation Results

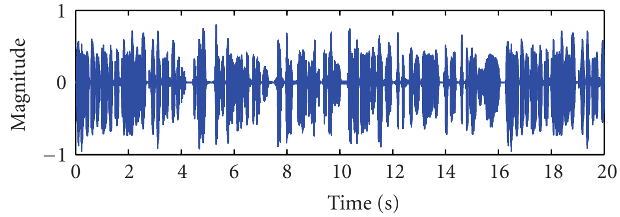
To evaluate the performance of the proposed algorithm, many computer simulations were conducted in the context of system identification. Four algorithms are compared in numerous simulations, APA, SPAPA, VSS-APA [24], and the proposed VSS-SPAPA. The unknown system $\mathbf{w}_{\text{opt}}$ is taken from a network echo path illustrated in Figure 1(a). Both $\mathbf{w}_{\text{opt}}$ and the adaptive filter $\mathbf{w}$ have same length, $N = 512$. For the proportionate algorithms, $\delta_\rho = 0.01$, $\rho = 1/N$. For VSS-SPAPA, $\lambda_1 = 1 - 1/2N$, $\lambda_2 = 1 - 1/4N$, $\alpha_{\min} = 0.005$. $\alpha_{\max} = 1.0$ is assigned because a large step size for SPAPA does not considerably improve its convergence speed but results in higher misalignment. The regularization parameters are chosen by $\epsilon = 10P\sigma^2_x$ for all the algorithms. The disturbance signal $v(t)$ is an independent white Gaussian noise. The convergence performance is evaluated using the normalized misalignment (in dB) defined by

$$10 \log_{10}\left(\frac{\left\|\mathbf{w}_{\text{opt}} - \mathbf{w}(t)\right\|_2^2}{\left\|\mathbf{w}_{\text{opt}}\right\|_2^2}\right). \qquad (25)$$

4.1. Simulations with Real Value of $\sigma^2_v(t)$. We first test the performance of the proposed VSS-SPAPA with the real value of $\sigma^2_v(t)$. The signal-to-noise rate (SNR) is adjusted to 20 dB for the simulation results illustrated. The misalignment of



(a)

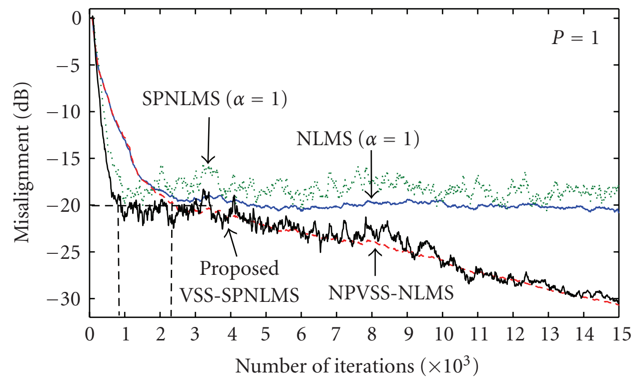


(b)

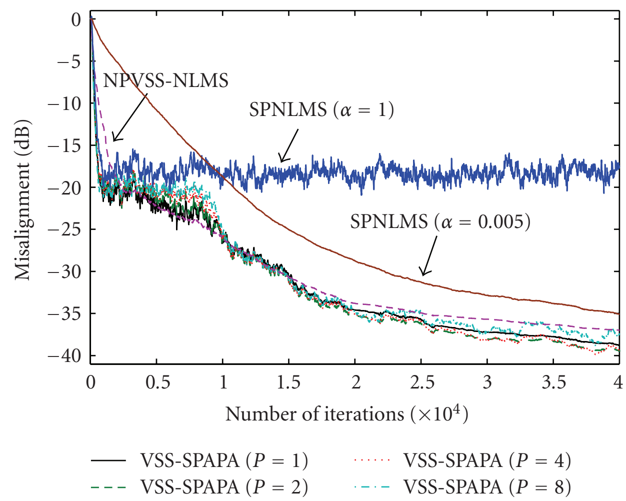
Figure 1: (a) A typical sparse network echo path used in the simulations. (b) A segment of speech signal with 8 k sampling rate used in the simulations.

the four algorithms is compared with three kinds of input signal: (a) white Gaussian noise signal, (b) highly colored signals generated by an AR(1) process, and (c) speech signals illustrated in Figure 1(b).

In the first set of simulations, the input sequence was zero-mean white Gaussian noise signal with $\sigma_x^2 = 1$. For APA and SPAPA, a constant step $\alpha = 1$ is assigned. Figure 2(a) compares convergence speed in the first 15000 iterations of the related algorithms. The projection order $P = 1$. Therefore, the related algorithms degrade into their corresponding NLMS versions. It can be seen that the SPNLMS algorithm converges faster than the conventional NLMS algorithm with same step size. We can also find that they have almost the same steady-state misalignment. The proposed VSS-SPNLMS algorithm achieves almost the same fast initial convergence as the SPNLMS algorithm. However, it can obtain lower steady-state misalignment; about 18 dB improvement can be observed in 4×10^4 iterations, as shown in Figure 2(b). To achieve this low level misalignment, the SPNLMS requires a very small step size. However, α is 0.005 in this case, whose convergence speed is greatly degraded. Although the NPVSS-NLMS algorithm in [21] can achieve almost the same low misalignment, its convergence speed is significantly lower than the proposed algorithm. It can be seen from Figure 2(a) that the proposed VSS-SPNLMS algorithm reaches −20 dB misalignment in approximately 900 iterations but the NPVSS-NLMS algorithm reaches that level in about 2200 iterations. Figure 2(b) illustrates the results of different projection order. It can be seen that in the case of white input signal, the increase in the projection order from 2 to 8 does not considerably improve the convergence speed of the proposed VSS-SPNLMS. This suggests that the control matrix $\mathbf{G}(t)$ determined by SPNLMS is nearly optimal for white input signal. The proposed VSS-SPNLMS is preferred to obtain optimal convergence speed and lower misalignment with least computation cost.



(a)



(b)

Figure 2: Misalignment of the algorithms with white Gaussian noise input. (a) $P = 1$. (b) Comparison of different projection order, $P = 1, 2, 4, 8$. SPNLMS and NPVSS-NLMS [21] are also illustrated.

In the second set of simulations, the input sequence $\{x(t)\}$ is an AR(1) process generated by filtering a zero-mean white Gaussian signal through a first-order system $G(z) = 1/(1 - 0.9z^{-1})$. Figure 3(a) illustrates the results with projection order $P = 2$. The proposed VSS-SPAPA achieves almost the same initial convergence speed with SPAPA, but it can reach a much lower steady-state misalignment—approximately 20 dB improvement can be observed in 5×10^4 iterations. Although VSS-APA can almost achieve this level of misalignment, its convergence speed is lower than the proposed VSS-SPAPA. Figure 3(b) shows the misalignment of the proposed algorithm of different projection order. In this case, when $P = 1$, all of the related algorithms present very low convergence speed. Their performance can be greatly improved by increasing $P = 2$. However, with the increase in the projection order from 4 to 8, the convergence speed of VSS-SPAPA does not increase further, while the convergence speed of APA and VSS-APA will improve with increased projection order from 1 to 8. The VSS-SPAPA with $P = 2$ is preferable in this case to obtain the best performance with a modest increase in computational complexity.

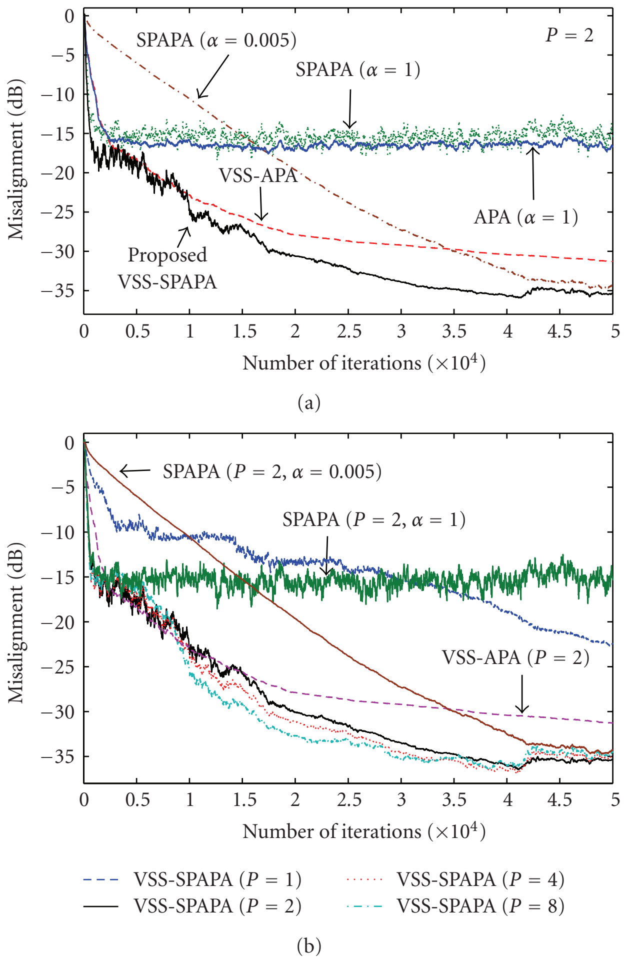


Figure 3: Misalignment of the algorithms with highly colored input generated by $G(z)$. (a) $P = 2$. (b) Comparison with different projection order, $P = 1$, 2, 4, 8. SPAPA and VSS-APA are also illustrated.

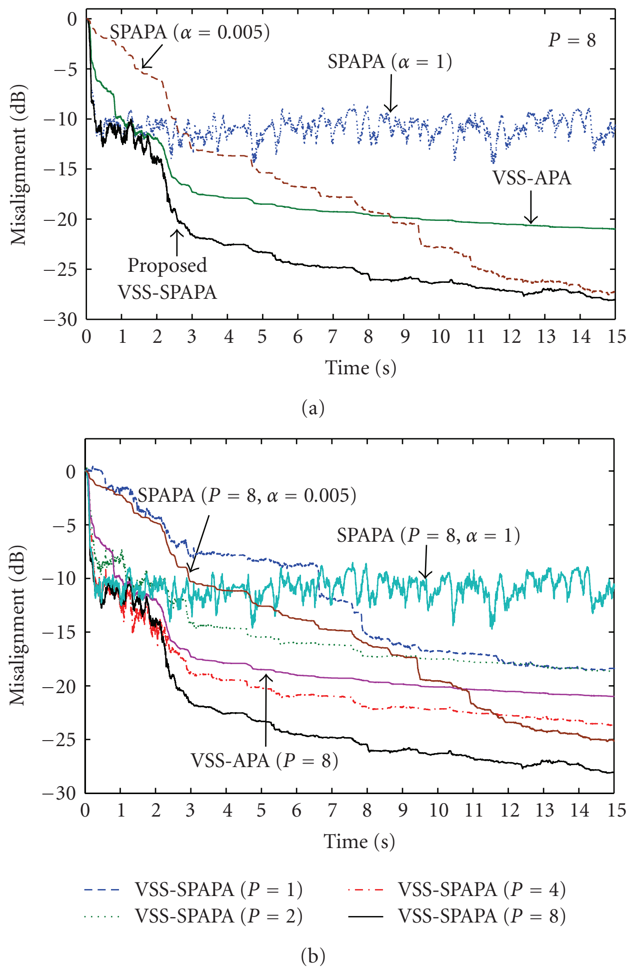


Figure 4: Misalignment of the algorithms with speech signal. (a) $P = 8$. (b) Comparison with different projection order, $P = 1$, 2, 4, 8. SPAPA and VSS-APA are also illustrated.

In the third set of simulations, the input is from a speech segment illustrated in Figure 1(b). The disturbance signal $\{v(t)\}$ is uncorrelated zero-mean white Gaussian noise with SNR= 20 dB. Figure 4(a) illustrates the result with projection order $P = 8$. The proposed VSS-SPAPA achieves almost the same fast initial convergence compared to the SPAPA, but it can achieve lower steady-state misalignment—-about 15 dB improvement can be observed in 15 seconds. In this case, VSS-APA cannot achieve this misalignment level (or it needs many more minutes to achieve it). The VSS-SPAPA outperforms VSS-APA both on convergence speed (approximately twice as fast) and on low misalignment (approximately 2 dB lower). Figure 4(b) compares the convergence of the proposed algorithm at different projection orders. In the case of speech signal input, with an increase in projection order from 1 to 8, the convergence speed of all the related algorithms was improved. Taking into account computational complexity, the performance of $P = 4$ is good enough in the case of speech signal input with a modest increase in computation load.

The tracking ability of the adaptive algorithms is important in a nonstationary environment where the unknown impulse response may suddenly change. In a network echo cancellation system, the echo path is subject to shift backward or forward as a result of delay jitter. Figure 5 illustrates the result of the tracking ability of the relevant algorithms with speech signal input and $P = 4$. The unknown impulse response suddenly shifted to the right by 12 samples. It can be seen from this figure that the proposed algorithm presents good tracking performance after the unpredicted change in the unknown impulse response. Furthermore, it outperforms its counterparts in low steady-state misalignment after reconvergence.

4.2. Simulations with Inaccurate Estimate of $\sigma_v^2(t)$. As discussed in the previous section, the estimated accuracy of $\hat{\sigma}_v^2(t)$ influences the convergence of VSS-APA and VSS-SPAPA. Figure 6 illustrates the simulation results of the relevant algorithms with AR(1) input signals in subplot (a), and speech input signals in subplot(b), respectively. We can

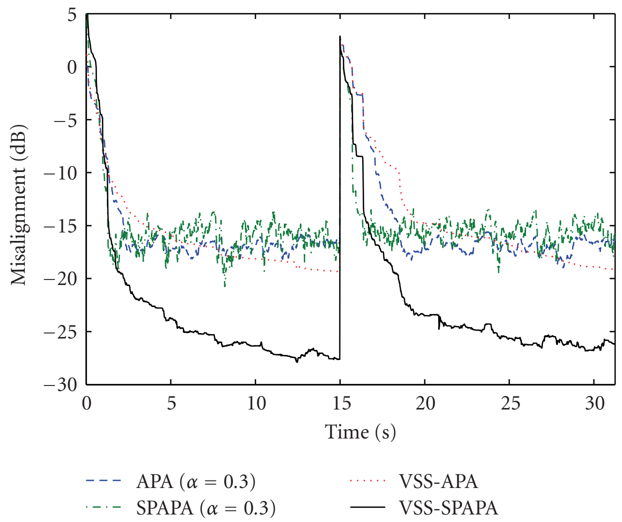


FIGURE 5: Comparison of tracking ability of the relevant algorithms with speech signals, $P=4$. The unknown impulse response changes after 15 seconds.

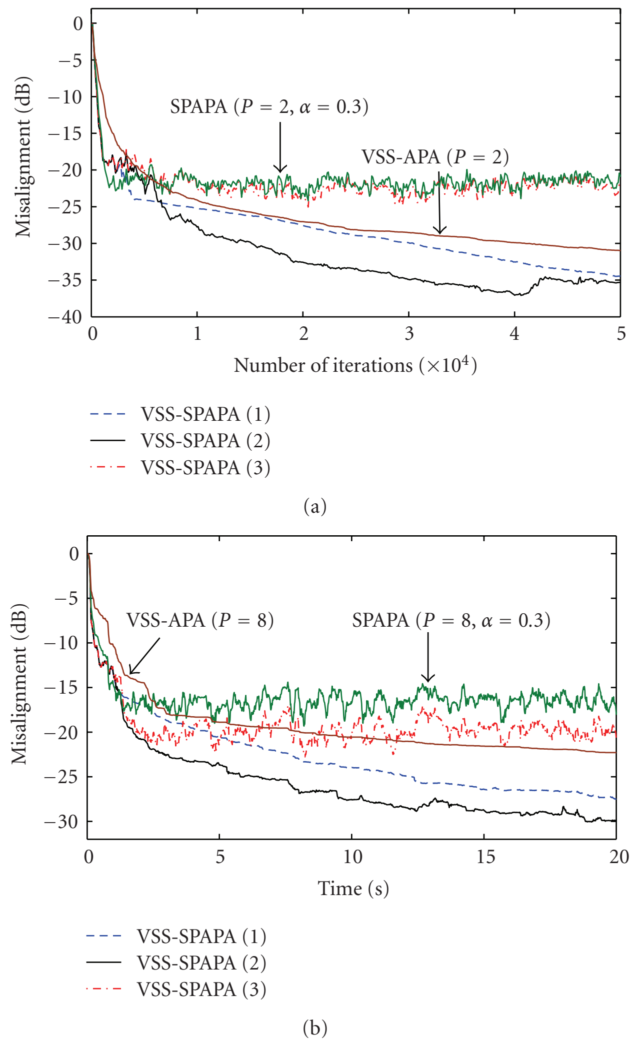


FIGURE 6: Misalignment of the relevant algorithms with inaccurate estimate of $\hat{\sigma}_v^2(t)$. (a) With highly colored input signals generated by $G(z)$. $P=2$. (b) With speech input signals, $P=8$. (1) $\hat{\sigma}_v^2(t) = 1.5\sigma_v^2(t)$, (2) $\hat{\sigma}_v^2(t) = \sigma_v^2(t)$, and (3) $\hat{\sigma}_v^2(t) = 0.8\sigma_v^2(t)$.

see that VSS-SPAPA with an accuracy of $\sigma_v^2(t)$ is best among the three cases: (1) the power estimate of the disturbance signal is greater than its real value, $\hat{\sigma}_v^2(t) = 1.5\sigma_v^2(t)$; (2) $\hat{\sigma}_v^2(t) = \sigma_v^2(t)$; (3) $\hat{\sigma}_v^2(t)$ is smaller than its real value, $\hat{\sigma}_v^2(t) = 0.8\sigma_v^2(t)$. In any case, VSS-SPAPA can maintain fast convergence with both AR(1) signals and speech signals. In case (1), a large estimate $\hat{\sigma}_v^2(t)$ will cause $\alpha_p(t)$ to be small and reach $\alpha_{\min}$ faster than in case (2). In any cases, $\alpha_{\min}$ warrants VSS-SPAPA to achieve low misalignment. It is obvious that this requires more iterations but is still faster than VSS-APA, as shown in the figure. However, if $\hat{\sigma}_v^2(t)$ is excessively smaller than its real value, the steady-state misalignment of VSS-SPAPA is relatively higher, as shown in the figure, because $\alpha_p(t)$ remains large in the steady state—around 0.25 in case (3). The VSS-SPAPA does not behave worse than the SPAPA even with a large estimate error of $\hat{\sigma}_v^2(t)$. It can tolerate more than +50% estimate error and about −20% estimate error of $\hat{\sigma}_v^2(t)$. The proposed VSS-SPAPA is robust to a relatively inaccurate estimate of $\hat{\sigma}_v^2(t)$.

If an estimate of $\sigma_v^2(t)$ is not available in practical application, it can be adaptively estimated according to (23). Figure 7 illustrates the results of the proposed VSS-SPAPA with this estimate method included. As discussed in the previous section, the problem with this adaptive estimate method is that the estimate is only effective when the adaptive filter has converged. Otherwise, the estimate will be very inaccurate and cause $\alpha_p(t)$ to be very small. Hence, the tracking ability of the proposed algorithms will be considerably worsened. So, the relevant algorithms are tested on the assumption that the unknown impulse response suddenly changes by shifting to the right 12 samples. Figure 7(a) is with the highly colored AR(1) input ($P=2$) and Figure 7(b) is with speech input signals ($P=8$). For SPAPA and APA, the constant step size is $\alpha = 0.3$, which is suitable for practical application. It can be seen that VSS-SPAPA can also quickly track the change in unknown impulse response and then achieve a lower misalignment after it has reconverged. It outperforms both the nonproportionate counterparts and the constant step-size algorithms. However, compared to the result with real value of $\hat{\sigma}_v^2(t)$, the steady-state misalignment of VSS-SPAPA with adaptive estimate of $\sigma_v^2(t)$ is worse than that with real value of $\sigma_v^2(t)$. For example, the misalignment of VSS-SPAPA in Figure 3 reaches −34 dB in 3×10^4 iterations, but in Figure 7(a) it only reaches about −30 dB. Figure 7(b) shows the result when the input is speech signal. The proposed VSS-SPAPA can achieve a lower steady-state misalignment than SPAPA, approximate 10 dB improvement was achieved. The steady-state misalignment of VSS-APA was the same as that of VSS-SPAPA, but it needs more time to reach that level. Compared to the scenario where the accurate estimate of $\sigma_v^2(t)$ is known, as shown in Figure 5 the steady-state misalignment in this scenario reaches only approximately −25 dB in 15 seconds. As expected, the reconvergence performances of

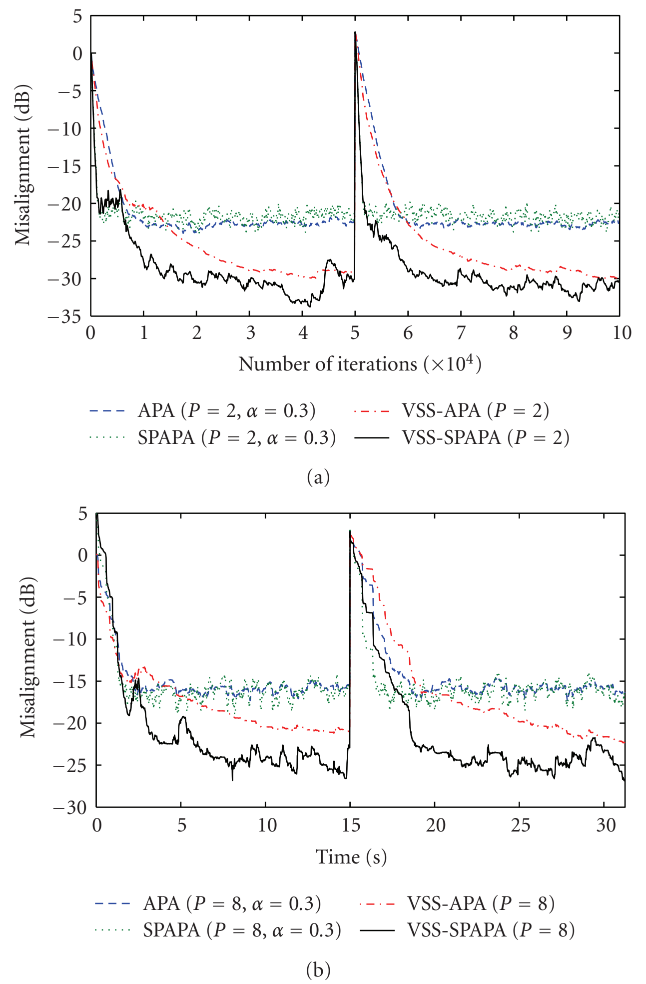


Figure 7: Misalignment of the algorithms with adaptive estimate of $\sigma_v^2(t)$. The unknown impulse response changes at 5×10^4 iteration. (a) With highly colored input generated by $G(z)$. $P = 2$. (b) With speech input signal, $P = 8$.

both VSS-APA and VSS-SPAPA are slower than their non-VSS counterparts during the period from 15 seconds to 20 seconds. Nevertheless, the VSS-SPAPA outperforms VSS-APA in convergence speed with almost the same steady-state misalignment.

In brief, the proposed VSS-SPAPA achieves faster convergence and lower misalignment than the conventional algorithms for the identification of sparse impulse response in the tested cases of different input signals.

5. Conclusions

We have proposed a method for introducing a variable step-size approach into the proportionate affine projection algorithm for identification of the sparse impulse response. The proposed algorithm can achieve not only very fast convergence but also relatively low misalignment. It is particularly efficient for highly colored input signals, such as speech. It does not require many parameter adjustment so it is easy to use in practical application. It only requires an estimate of $\sigma_v^2(t)$, the power level of the disturbance signals. If this estimate is not available, an adaptive estimate method is applicable with only a little performance loss. Simulations show that the proposed VSS-SPAPA outperforms the conventional adaptive algorithms for the identification of sparse impulse response.

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